Time-varying overreaction of diagnostic expectations

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Abstract

Diagnostic expectation is a non-rational expectation framework that representativeness heuristics distort agents' belief and make agents overreact to observed signals. It helps explain systematic analysist' forecast error and revisions. It is common that the strength of representativeness is time-invariant. However, psychological heuristics is mental short-cut to reduce the difficulties associated with complex decision making. The benefits and incentives would be different in each situation. Therefore, we examine how representativeness strength varies in time. We use TOPIX futures prices as indirect forecast data. We split the samples to five-year subsamples and run SMM to estimate the representativeness strength. we find that investors overreact to signals in general, but the strength varies in time significantly and it is not monotonous. We also find that investor overreaction is strong when the signal is informative. If the strength of representativeness follows an AR(1) process, it is less persistent when the signal is not informative. It implies that agents suddenly overreact to information or turn to rational expectation. Our results suggest that there is second way of interpretation of "context-dependent" of distorted beliefs due to psychological heuristics. This analysis is related to the selective memory literature and suggests that the weights or order of memory recall can vary in time depending on the market conditions.

Keywords: Diagnostic expectations; overreaction; representativeness heuristics

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EFM Codes: 720; 310; 570

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1 Introduction

To solve the excess volatility puzzle and equity premium puzzle, many researchers propose the additional required return. In recent years, some studies take non-rational expectation approach to solve these puzzles. This approach uses psychological heuristics as ingredients for expectation framework. Bordalo *et al.* (2018) and Bordalo *et al.* (2019) use representativeness heuristics and show that investors distort their beliefs from rational expectation by inflating probability of representative events or states.

When decision maker observes new information, states or events whose probability increases the most come to mind first and they are oversampled in mind. As a result, decision maker overreacts to observed information by overestimating the probability of events which is more likely to occur. The extent of overreaction and overestimated events depend on the prior beliefs.

This distorted expectation, called diagnostic expectation, shares some features with extrapolative expectation, which agents systematically assume that realized changes keep occurring in the future (Barberis *et al.* 2015, Barberis *et al.* 2018). However, extrapolative expectation is backward looking whereas diagnostic expectation is forward looking because agents compare the prior probability with posterior probability and overweight probability of representative states. Therefore, latter expectation is context-dependent framework.

Most literature of diagnostic expectations assumes that the strength of representativeness which distorts subjective probability is time-invariant. Given that diagnostic expectation is context-dependent, it is not clear whether impacts of representativeness is also context-dependent. Psychological heuristic is mental shortcut for quick decision making to avoid costly complex problems. It is possible that the unconscious incentives of heuristics depend on the situations agents face and the impact of heuristics is context-dependent. To investigate the time variation of psychological influences, this paper aims to examine whether investor overreaction due to diagnostic expectation fluctuates in time.

We use the TOPIX futures price as market expected value of TOPIX, one of the most famous indexes in Japanese market. We split the data to subsamples and run the simulation to estimate the strength of representativeness in each period. We use the SMM following

Bordalo *et al.* (2019) methodology, but target data is replaced to Japanese data. The sample period is from January 1998 to December 2022. The subsample has five years length so that we have five subsamples.

Our results are followings. First, we run all samples simulation to compare the Japanese market to US market. In terms of parameters of fundamental, we obtain similar relationship between fundamental and transitory shocks. We find that process of fundamental is persistent and its variance is higher than transitory shock. Looking to diagnostic parameter, we obtain similar value to US analysis; Bordalo *et al.* (2019) observe 0.9 whereas our result is 1.06. Both values imply strong overreaction to current price movement.

Second, we split sample and run same simulation to compare the parameters. In terms of parameters of fundamental, our results show that subsamples after global financial crisis, between 2008 and 2017 called post crisis samples, have less persistent process of fundamental but the other samples have close to all samples analysis. Post crisis samples have lower variance of fundamental than one of transitory shock, implying that TOPIX price is less informative about fundamental.

For diagnostic parameter, we observe that post crisis samples have less than one whereas others have over 1.3. There is a clear gap. This result implies that Japanese market overreact to news very strongly when news is informative. In contrast, during 2008 and 2017, TOPIX prices are not as informative as other periods, and Japanese market relatively weakly overreact to news. Notice that even market is not informative and the representativeness impact is not as high as other periods in post crisis samples, diagnostic Kalman gain is over 0.5, suggesting that investors still react to price movements as if it is informative.

Third, we assume that strength of representativeness follows AR(1) process like fundamental and run the simulation. This setting allow us to examine the case where market participants' overreaction is varying. We observe that parameters of fundamental do not vary in subsamples. It suggests that this setting is more reliable than former simulation to capture macroeconomic structure.

Persistence of diagnostic parameter is not consistent between all samples and subsamples. All samples and post crisis samples show weak persistence whereas the other samples show that it is persistent. The standard deviation of diagnostic strength process is ranged in about 0.2 and 0.38. All samples and post crisis samples have lower volatility. Standard deviation of 0.2 implies that average change of overreaction is about 20%. We also find that volatility of diagnostic strength is high and persistence of diagnostic strength is low when news is informative. This is consistent with second finding in a manner that investors tend to overreact more when market is informative.

We have mainly two contributions. First, we add non-US analysis by using Japanese market data to non-rational expectation literature. Most papers in this field conduct empirical analysis with US analyst data. Greenwood and Shleifer (2014) show the survey data of forecasts of many types of US investors. Bordalo *et al.* (2019) study US analysts forecasts and explain their systematic forecast errors. In contrast, we use future prices of Japanese market index. Although it is not direct data of investors' forecasts, market prices reflect aggregated investors' belief in general. Our result suggests that not only professional analysts, but all market participants have overreaction tendency. This is in line with literature which show that many types of agents hold distorted beliefs (Andre *et al.* 2022, Bordalo *et al.* 2020a).

Second, we offer news insights of time-varying characteristics of diagnostic expectations. Our result shows that the strength of representativeness varies significantly and suggests that this varying feature and persistence of this strength might be correlated with market informativeness. Afrouzi *et al.* (2023) show that agents tend to overreact to further future forecasts of lower persistent process. Our result is closed to their implications. In addition, the asymmetry of overreaction impacts between informative period and less-informative period might contribute to different behavior of economic agents in boom and burst periods. Maxted (2023) and Krishnamurthy and Li (2020) uses diagnostic expectations and examines the risk tolerance of banks before and after boom phase. Our result might enhance their arguments.

This paper is organized as follows. Section two summarizes the existing literature of diagnostic expectations. Section three describes the framework of diagnostic expectations and section four describes the simulation methodology. Section five shows the result of simulation. Section six concludes.

2 Prior literature

In the asset pricing field, there are puzzles. The stock returns are more volatile than justified by their dividend flows (Shiller *et al.* 1981). The required return is higher than expected (Campbell and Shiller 1988). Many researchers propose the variety of required returns; time varying risk preferences (Campbell and Cochrane 1999), and long run risk or disaster risk model (Bansal and Yaron 2004, Barro 2009). Firm characteristics-based factor models are also proposed (Fama and French 2015). These models assume that investors have rational expectations.

In recent years, some studies take another approach: in stead of rational expectations, investors' expectations are shaped by investor sentiment or psychological heuristics. Barberis *et al.* (1998) propose the investor sentiment driven model and Barberis and Shleifer (2003) consider style investing. Gennaioli and Shleifer (2010) take account of representativeness heuristics, the tendency that people overweight the probability of an event when it is representative of characteristics to its parent population (Kahneman and Tversky 1972).

Based on the psychological background, Bordalo *et al.* (2018) propose the diagnostic expectations. In this framework, expectations are formed by representativeness heuristics. People overestimate the probability of event which is more likely to occur under the parent population. For example, if investor receive the positive signal, rational investors update their beliefs according to the Bayes' rule, but diagnostic investors, who has diagnostic expectation, overestimate the high productivity state because such state is representative to positive signals. Therefore, diagnostic investors overreact to positive signals and their expectation becomes more optimistic.

Diagnostic expectations make investor overreact to current news, and their beliefs more volatile than rational expectations. These features can explain the survey results of Greenwood and Shleifer (2014) which states that forecasts of many types of agents in US exhibit extrapolative and volatile characteristics. Bordalo *et al.* (2018) show that credit spreads are excessively volatile, overreact to news, and entail predictable reversals. Bordalo *et al.* (2021b) show that when productivity growth decreases, credit spreads excessively increase. Bordalo *et al.* (2022) shows that overreaction due to diagnostic expectations generates the predictable boom-bust cycles. Bordalo *et al.* (2020c) show that in heterogeneous agent model, diagnostic expectations generate individual investors' overreaction, but consensus beliefs exhibit underreaction to news because each investor observes different information. d'Arienzo (2020) applies to affine term-structure model and shows that long-term interest rates are excessively sensitive to news and excessively volatile because it has higher uncertainty than short-term and it makes any signals more informative.

Diagnostic expectations are used to explain the consumption behavior. L'Huillier *et al.* (2021) show that consumption also overreacts to supply shocks. Bianchi *et al.* (2021a) explain the persistent and hum-shaped boom-bust cycle of consumptions. Bianchi *et al.* (2021b) show that consumption becomes time-inconsistent when reference point is not recent.

Bordalo *et al.* (2019) apply diagnostic expectations to stock return analysis and show that diagnostic expectations can explain the analysists forecasts' systematic forecast errors and revisions. Bordalo *et al.* (2020b) show that systematic overreaction generates the price reversals and excess stock market volatility. Bordalo *et al.* (2021a) show that price overreaction leads to endogenous bubbles and crash.

Krishnamurthy and Li (2020) introduce diagnostic expectations into frictional financial intermediation models and show that banks with diagnostic expectations have higher risk tolerance, decrease the risk premia and increase the credit before crisis. Maxted (2023) shows that disappointment after boom due to excessive optimism of diagnostic expectations make banks tighten their lending, leading to crisis.

In line with literature, we contribute in two ways. First, we conduct non-US analysis. Most empirical analysis or calibrations builds on US data. However, psychological background of diagnostic expectations is not exclusive for US. We use Japanese data and estimate the parameters of diagnostic expectations. We complement this field by adding the Japanese market analysis.

Second, existing literature assumes that the extent of representativeness, which is the source of belief distortion in diagnostic expectations, is time-invariant. Given that diagnostic expectation is context-dependent framework, it should be studied whether representativeness strength is also context-dependent and time-varying. For example, Chen and Sauer (1997) show that return of contrarian portfolio, buying losers and selling winners, is not always positive, suggesting that stock market overreaction varies in time. Therefore, we split samples and examine the diagnostic property. We offer new insights of diagnostic property.

This paper is also related to the field of selective memory. In selective memory literature, people recall the memory from their database and the order of recalling or weights of memory make beliefs overreaction to recent signals (Bordalo *et al.* 2020a, Nagel and Xu 2022). Since this paper examine the changes in representativeness strength in time, our paper corresponds to the time-varying properties of memory recalling order or weights.

3 Model: Diagnostic expectation

In this section, we explain diagnostic expectation which is developed by Bordalo *et al.* (2018) and Bordalo *et al.* (2019). It is assumed in Bordalo *et al.* (2019) that the fundamental of economy follows the law of motion

$$f_t = af_{t-1} + \eta_t \tag{1}$$

where $a \in [0,1]$ and $\eta_t \sim N(0, \sigma_\eta^2)$ is an iid normally distributed shock. It is assumed that investors cannot observe f_t directly. Instead, they observe x_t given by

$$x_t = bx_{t-1} + f_t + \epsilon_t \tag{2}$$

where $in[0,1]$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ is an iid normally distributed shock. We assume stationarity by imposing $b \leq a$.

Rational investors update their beliefs according to the Kalman filter to infer the current fundamental.

$$E[f_t \mid x_t] = \hat{f}_t = a\hat{f}_{t-1} + K(x_t - bx_{t-1} - a\hat{f}_{t-1}) \tag{3}$$

where $K \equiv (a^2\sigma_f^2 + \sigma_\eta^2)/(a^2\sigma_f^2 + \sigma_\eta^2 + \sigma_\epsilon^2)$ is the signal-to-noise ratio[1].

[1] In steady state, te variance of fundamental is given as the solution to $a^2\sigma_f^4 + \sigma_f^2[\sigma_\eta^2 + (1-a^2)\sigma_\epsilon^2] - \sigma_\eta^2\sigma_\epsilon^2 = 0$.

Bordalo *et al.* (2018) and Bordalo *et al.* (2019) propose that investors' beliefs are distorted by the representativeness heuristics. It is argued that agents overestimate the probability of events which is a representative or typical of a class (Kahneman and Tversky 1972). Using the measure of the representativeness proposed by Gennaioli and Shleifer (2010), diagnostic expectations are formed by the representativeness-distorted density

$$h^{\theta}(f_t \mid x_t) = h(f_t|x_t)[h(f_t \mid x_t)/h(f_t \mid x_{t-1})]^{\theta} Z \tag{4}$$

where $h(f_t \mid x_t)$ is a rational conditional density, $\theta \geq 0$ is a parameter of representativeness severity, and Z is a constant ensuring that $h^{\theta}(f_t \mid x_t)$ integrates to one. With $\theta = 0$, there is no distortions, and it becomes rational density.

This setup shows that investors compare the current true density $h(f_t \mid x_t)$ with past true density $h(f_t \mid x_{t-1})$ and overestimate (underestimate) the probability if it is more (less) likely to happen when new information is observed. For example, if there is a positive surprise, then the fundamental is more likely to be high, so investors overestimate such states and overreact to positive surprise. Diagnostic expectations are formed by following distorted Kalman filter.

$$E^{\theta}[f_t \mid x_t] = \hat{f}_t^{\theta} = a\hat{f}_{t-1} + K(1+\theta)(x_t - bx_{t-1} - a\hat{f}_{t-1}) \tag{5}$$

Bordalo *et al.* (2019) also show that expectations of long-term growth of x_t (LTG) are characterized by mean reversion as well as fundamental and signal.

$$LTG_{t,h} = E^{\theta}[x_{t+h} - x_t \mid x_t] = -(1-b^h)x_t + a^h \frac{1-(b/a)^h}{1-(b/a)} \hat{f}_t^{\theta} \tag{6}$$

LTG becomes high when there is a positive news and $\hat{f}_t^{\theta}$ increases. Because $\hat{f}_t^{\theta} \geq \hat{f}_t$ if there is a positive shock, diagnostic investors become optimistic.

4 Simulation

Bordalo *et al.* (2019) use the SMM to estimate the diagnostic parameter θ. They set the parameters $(a, b, \sigma_\eta, \sigma_\epsilon, \theta)$ to match the autocorrelation (ρ_l) of signal (x_t) at lags $l = 1, 2, 3, 4$

years, and the coefficients (γ_k) of forecast revision ($LTG_{t,h} - LTG_{t-k,h}$) to forecast errors ($(x_{t+h} - x_t) - LTG_{t,h}$) for $k = 1, 3$ years.

The purpose of this paper is to study the time variation of investor overreaction. According to diagnostic expectations, overreaction is driven by the diagnostic parameter (θ). Therefore, we aim to analyze whether the diagnostic parameter varies and the extent of such variation.

Bordalo *et al.* (2019) uses EPS as signal of fundamental (x_t) and analyst forecasts of EPS for LTG. In contrast, we use the TOPIX market prices as a signal of fundamental and TOPIX futures prices for LTG.

We run same SMM analysis for all sample period between January 1998 and December 2022. In addition, we also split the sample to 5 years and run the same SMM to obtain the diagnostic parameters for subsample period. It ensures how the extent of investors' overreaction varies.

Bordalo *et al.* (2019) assume the constant diagnostic parameter, but it is not clear whether it also evolves like fundamental. To consider such situation, we assume the time-varying diagnostic parameter and estimate its law of motion.

$$\theta = \max(\theta_{t-1} + \xi_t, 0) \tag{7}$$

where $\xi \sim N(0, \sigma_\xi^2)$ is an iid normally distributed shock and max operator exclude the negative θ case where investors oppositely react to news; when there is a good surprise, they expect that fundamental goes bad. This formulation implies that the extent of investors' overreaction depends on previous one and they can be rational even they are not in previous time.

5 Results

5.1 Target coefficients

Before estimating the diagnostic parameter, we need a measure of overreaction at each time. Following Bordalo *et al.* (2019), we run the Coibion and Gorodnichenko (2015) test.

$$x_{t+h} - x_t - LTG_{t,h} = \alpha + \gamma(LTG_{t,h} - LTG_{t-k,h}) + e_{t+h} \tag{8}$$

Since diagnostic investors systematically overreact to current surprises, the expected sign of γ is negative for $\theta > 0$ and zero for $\theta = 0$ (corresponding to rational investors).

We consider that the price of futures whose contract month is 1 month ahead is the expected value of TOPIX price 1 month ahead. Because contract month of TOPIX futures is every 3 months, we fix the revision interval (k) to 3 months.

Table 1 shows the results of CC regression. We find that there is a negative relationship between forecast revision over past 3 months and forecast error over 1 month. This is also found in relation of forecast error over 4 months. Except for 2 and 3 months forecast error, positive surprises drive higher growth expectations, but it is higher than realized growth on average.

5.2 Fixed diagnostic parameter

Bordalo *et al.* (2019) set the six parameters $(a, b, \sigma_\eta, \sigma_\epsilon, \theta, s)$ where s is the reference time of the lagged expectations. Due to the data characteristics, we set s=3 so that our parameters are $(a, b, \sigma_\eta, \sigma_\epsilon, \theta)$. For every combination of parameters, we simulate a time series of fundamental (f_t) and TOPIX prices (x_t) and calculate the associated diagnostic expectations $(\hat{f}_t^\theta)$ about fundamental. Then, we compute the autocorrelation of TOPIX prices $\hat{\rho}_l = cov(x_t, x_{t-l})/Var(x_t)$ for $l = 1, 2, 3, 4$ months. In addition, we regress the forecast error $(x_{t+h} - x_t - LTG_{t,h})$ on forecast revision $(LTG_{t,h} - LTG_{t-3,h})$ for $h = 1, 4$ months to get coefficients $\hat{\gamma}_1$ and $\hat{\gamma}_4$. This yields the vector

$$v(a, b, \sigma_\eta, \sigma_\epsilon, \theta) = (\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4, \hat{\gamma}_1, \hat{\gamma}_4) \tag{9}$$

We repeat this procedure for each parameter combination. Each parameter range is defined by $a, b \in [0, 1]$, $\sigma_\eta, \sigma_\epsilon \in [0, 0.5]$, and $\theta \in [0, 2]$. a, b, and θ are defined in steps of 0.1 and σ_η and σ_ϵ are defined in steps of 0.025.

After all, we estimate the parameters by picking the combination that minimizes the Euclidean distance loss function

$$l(v) = ||v - \bar{v}|| \tag{10}$$

where $\bar{v}$ is the vector of target moments estimated from the original data. The target vector in our analysis is summarized in table 2. Our result shows that autocorrelation of 1 month is about 0.83-0.97. This suggests that autocorrelation of TOPIX decreases after the global financial crisis. Notice that the length of subsample is 5 years. Subsample of 2008 and 2018 contains the global financial crisis and COVID outbreak. The coefficient of forecast revision to forecast error in one month is positive for these subsamples. This implies that initial reaction during these uncertain situations was not enough so that subsequent price adjustments occur.

Table 3 shows the mean and standard deviation of estimates of model parameters for all sample and subsamples which minimize the Euclidean loss function across 30 independent simulations. Estimated persistence parameter of fundamental (a) is 0.87 for all sample, but it drops to 0.65 for 2008 and 2013 subsamples. These subsamples have higher standard deviations for a, implying that there would be difficulties for simulation during these periods. The volatility of fundamental (σ_η) is higher than the volatility of transitory (σ_ϵ) for all sample, but it is less for post crisis samples (2008 and 2013 subsamples). Lower volatility of transitory shock implies that the change of signals is more likely to originate from the unobserved fundamental. Therefore, we observe Kalman gain (K) is above 0.5 for all sample, but not for post crisis samples.

Most interested parameter in this analysis is the diagnostic parameter of θ. There are two key findings. First, our estimates show the strong diagnostic effects. θ is about or over one for most of the samples except 2013 subsample which is the lowest value of 0.58. The highest mean value of θ is 1.83 for 1998 subsample. Since diagnostic investors react to surprises by $(1+\theta)K$, if θ is one, then they react to news as if signal-to-noise ratio is doubled. For example, for all sample, Kalman gain is 0.7, but diagnostic Kalman gain is 1.45, suggesting that they excessively overreact to new surprises.

When Kalman gain and diagnostic parameter are small, impact of diagnostic expectation is not as huge as expected to cause the overreaction because small Kalman gain implies the uninformative signal and small θ is relatively small overreaction. However, our diagnostic Kalman gain for post crisis samples, which have small Kalman gain and θ, are above 0.5. It suggests that even observed signal has more noise than fundamental, diagnostic investors systematically think it is informative and try to extract the information about fundamental.

Second, diagnostic parameters vary significantly across time. The lowest value is 0.58 and the highest value is 1.83, more than 3 times of lowest value. Diagnostic parameter decreases in post crisis samples, but it recovers to 1.34 in 2018 subsample. It does not show the monotonic trend. Even though the standard deviation of θ is high, this variation suggests that the extent of representativeness which is the source of diagnostic expectations would not stay constant and vary in time.

5.3 Time-varying diagnostic parameter

In previous analysis, we observe that θ varies in time. Its variation seems not to be monotonical change. The level of θ is 1.83 in 1998 subsample, whereas it is 0.58 in 2013 subsample, which is less than one third of the highest value. Then, it recovers to 1.34 in 2018 subsample. In this section, we study the case where θ is also formed by AR(1).

In this setup, estimated parameters are $(a, b, c, \sigma_\eta, \sigma_\epsilon, \sigma_\xi)$. We assume that θ follows the equation (7).

As in the previous analysis, we simulate a time series of fundamental (f_t), TOPIX prices (x_t), and the diagnostic parameter (θ_t). Then we compute the autocorrelation of TOPIX prices and coefficients of forecast revision to forecast errors. Parameters of θ_t are defined by

$c \in [0, 1]$ and $\sigma_\xi \in [0, 0.5]$ with steps of 0.1 and 0.025. We pick the parameter combination which minimize the Euclidean loss function. Notice that the target vector is same to previous one in table 2. We set the initial value of c is 0, corresponding to the rational expectation case.

Table 4 shows the mean and standard deviation of estimates of model parameters across 30 independent simulations. In this analysis, fundamental is estimated more persistent than previous one. It is around 0.9. Looking at volatility of fundamental (σ_η), it always has higher than the volatility of transitory shock (σ_ϵ). Therefore, Kalman gain is above 0.5 for all time and the observed signal is informative.

The most interested parameter in this analysis is the persistence (c) and the volatility (σ_ξ) of θ. First, we find the mixed result in terms of persistence. Estimated parameter for all sample is 0.39, which is less persistent. However, it is above 0.67 for majority of the subsamples. Notice that all samples have similar standard deviations, but the level is higher than other parameters like a or b. Higher persistence implies that the extent of representativeness gradually change so that the overreaction cyclically occurs. If persistence of θ is low, people suddenly overreact to news or turn to rational expectations. It may be caused other hidden variables. In our analysis, latter case occurs during 2008 and 2017, the periods of global financial crisis and COVID outbreak.

Second, our estimated volatility (σ_ξ) of θ is 0.21 for all sample, varying between 0.2 and 0.38. Given that diagnostic Kalman gain is calculated by $(1 + \theta)K$, average change of overreaction is about 20%. We also observe that σ_ξ is high when Kalman gain is high. If σ_ξ correlates to Kalman gain positively, overreaction due to the diagnostic expectation are more likely to occur when the signal is more informative.

6 Conclusion

Diagnostic expectation is a framework which investors overestimate the representative states and overreact to observed signals. We investigate how this belief distortion are different in each subsample and running simulation to estimate the diagnostic parameters in Japanese data. By conducting the non-US analysis, We enrich diagnostic literature.

Our result is consistent with US analysis for all sample analysis. However, splitting the sample and corresponding simulation shed light on behavior of investor overreaction. We find that investors strongly overreact to news or signals in majority periods, but in periods after global financial crisis between 2008 and 2017 investor overreaction is weakened. Signals during latter period have lower informativeness. It suggests that overreaaction is more likely to occur when market is informative. Notice that even low informative periods, diagnostic Kalman gain is over 0.5, imlying that investors reeact to signals as if it is informative.

We also find that in time-varying diagnostic strength model, these period entails lower persistence and higher volatility of strength process. It suggests that investors tend to stay rational expectations and overreact to signals relatively less during these periods. This result offers new insights for investor sentiment analysis. Investor sentiment might be context-dependent.

References

Afrouzi, H., Kwon, S. Y., Landier, A., Ma, Y. and Thesmar, D. (2023). Overreaction in expectations: Evidence and theory. *The Quarterly Journal of Economics*, p. qjad009.

Andre, P., Pizzinelli, C., Roth, C. and Wohlfart, J. (2022). Subjective models of the macroeconomy: Evidence from experts and representative samples. *The Review of Economic Studies*, **89** (6), 2958–2991.

Bansal, R. and Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *The journal of Finance*, **59** (4), 1481–1509.

Barberis, N., Greenwood, R., Jin, L. and Shleifer, A. (2015). X-capm: An extrapolative capital asset pricing model. *Journal of financial economics*, **115** (1), 1–24.

—, —, — and — (2018). Extrapolation and bubbles. *Journal of Financial Economics*, **129** (2), 203–227.

— and Shleifer, A. (2003). Style investing. *Journal of financial Economics*, **68** (2), 161–199.

—, — and Vishny, R. (1998). A model of investor sentiment. *Journal of financial economics*, **49** (3), 307–343.

Barro, R. J. (2009). Rare disasters, asset prices, and welfare costs. *American Economic Review*, **99** (1), 243–264.

Bianchi, F., Ilut, C. L. and Saijo, H. (2021a). *Diagnostic business cycles*. Tech. rep., National Bureau of Economic Research.

—, — and — (2021b). Implications of diagnostic expectations: Theory and applications.

Bordalo, P., Coffman, K., Gennaioli, N., Schwerter, F. and Shleifer, A. (2020a). Memory and representativeness. *Psychological Review*, **128** (1), 71.

—, Gennaioli, N., Kwon, S. Y. and Shleifer, A. (2021a). Diagnostic bubbles. *Journal of Financial Economics*, **141** (3), 1060–1077.

—, —, La Porta, R. and Shleifer, A. (2020b). *Belief overreaction and stock market puzzles*. National Bureau of Economic Research.

—, —, Ma, Y. and Shleifer, A. (2020c). Overreaction in macroeconomic expectations. *American Economic Review*, **110** (9), 2748–2782.

—, —, Porta, R. L. and Shleifer, A. (2019). Diagnostic expectations and stock returns. *The Journal of Finance*, **74** (6), 2839–2874.

—, — and Shleifer, A. (2018). Diagnostic expectations and credit cycles. *The Journal of Finance*, **73** (1), 199–227.

—, — and — (2022). Overreaction and diagnostic expectations in macroeconomics. *Journal of Economic Perspectives*, **36** (3), 223–244.

—, —, — and Terry, S. J. (2021b). *Real credit cycles*. Tech. rep., National Bureau of Economic Research.

Campbell, J. Y. and Cochrane, J. H. (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of political Economy*, **107** (2), 205–251.

— and Shiller, R. J. (1988). The dividend-price ratio and expectations of future dividends and discount factors. *The Review of Financial Studies*, **1** (3), 195–228.

Chen, C. R. and Sauer, D. A. (1997). Is stock market overreaction persistent over time? *Journal of Business Finance & Accounting*, **24** (1), 51–66.

Coibion, O. and Gorodnichenko, Y. (2015). Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review*, **105** (8), 2644–2678.

d'Arienzo, D. (2020). Maturity increasing overreaction and bond market puzzles. *Available at SSRN 3733056*.

Fama, E. F. and French, K. R. (2015). A five-factor asset pricing model. *Journal of financial economics*, **116** (1), 1–22.

Gennaioli, N. and Shleifer, A. (2010). What comes to mind. *The Quarterly journal of economics*, **125** (4), 1399–1433.

Greenwood, R. and Shleifer, A. (2014). Expectations of returns and expected returns. *The Review of Financial Studies*, **27** (3), 714–746.

Kahneman, D. and Tversky, A. (1972). Subjective probability: A judgment of representativeness. *Cognitive psychology*, **3** (3), 430–454.

Krishnamurthy, A. and Li, W. (2020). *Dissecting mechanisms of financial crises: Intermediation and sentiment*. Tech. rep., National Bureau of Economic Research.

L'Huillier, J.-P., Singh, S. R. and Yoo, D. (2021). Incorporating diagnostic expectations into the new keynesian framework. *Available at SSRN 3910318*.

Maxted, P. (2023). A macro-finance model with sentiment. *Review of Economic Studies*, p. rdad023.

Nagel, S. and Xu, Z. (2022). Asset pricing with fading memory. *The Review of Financial Studies*, **35** (5), 2190–2245.

Shiller, R. J. *et al.* (1981). Do stock prices move too much to be justified by subsequent changes in dividends?

Table 1. Result of Coibion and Gorodnichenko (2015) test.

	coeffs	stand. err	p value
h=1	-2.127	0.614	0.001
h=2	1.597	1.733	0.360
h=3	0.185	1.608	0.909
h=4	-0.328	0.103	0.002
h=5	-0.191	0.104	0.071
h=6	-0.274	0.107	0.013

This table shows the regression coefficient of forecast errors on forecast revision. We fix the forecast revision intercal to 3 month.

Table 2. Target coefficient values.

	all sample	1998	2003	2008	2013	2018
ρ_1	0.97	0.928	0.948	0.864	0.832	0.859
ρ_2	0.944	0.856	0.897	0.732	0.718	0.786
ρ_3	0.916	0.771	0.837	0.63	0.615	0.685
ρ_4	0.887	0.69	0.769	0.517	0.553	0.631
γ_1	-2.177	-4.839	-6.668	2.669	-4.346	2.95
γ_4	-0.532	-0.301	0.486	-11.537	-3.883	-1.403

ρ_l is a autocorrelation of TOPIX for $l = 1, 2, 3, 4$ months. γ_h is a regression coefficient of CG test for $h = 1, 4$ months. These are target vectors for simulations.

Table 3. Estimated coefficients of simulation.

	all sample	1998	2003	2008	2013	2018
a	0.873	0.86	0.857	0.653	0.637	0.893
	(0.052)	(0.05)	(0.05)	(0.296)	(0.318)	(0.025)
b	0.47	0.68	0.687	0.363	0.123	0.537
	(0.274)	(0.061)	(0.09)	(0.243)	(0.05)	(0.138)
σ_η	0.344	0.42	0.387	0.148	0.213	0.408
	(0.11)	(0.08)	(0.096)	(0.12)	(0.16)	(0.098)
σ_ϵ	0.255	0.158	0.158	0.321	0.352	0.217
	(0.183)	(0.135)	(0.125)	(0.135)	(0.112)	(0.137)
θ	1.063	1.833	1.633	0.983	0.583	1.34
	(0.798)	(0.188)	(0.47)	(0.597)	(0.484)	(0.521)
Kalman gain(K)	0.704	0.885	0.869	0.256	0.347	0.807
diagnostic K	1.452	2.508	2.289	0.507	0.55	1.889

This table shows the mean value and standard deviation of estimated coefficients. Standard deviation is inside the brackets. Kalman gain is calculated by $K \equiv (a^2\sigma_f^2 + \sigma_\eta^2)/(a^2\sigma_f^2 + \sigma_\eta^2 + \sigma_\epsilon^2)$ and diagnostic Kalman gain (diagnostic K) is calculated by $(1+\theta)K$. If Kalman gain or diagnostic Kalman gain is over one, investors overreact observed signals.

Table 4. Estimated coefficients of simulation.

	all sample	1998	2003	2008	2013	2018
a	0.84	0.897	0.9	0.9	0.9	0.877
	(0.056)	(0.018)	(0.0)	(0.0)	(0.0)	(0.043)
b	0.207	0.67	0.693	0.41	0.163	0.48
	(0.146)	(0.121)	(0.105)	(0.099)	(0.085)	(0.089)
c	0.393	0.74	0.767	0.463	0.483	0.673
	(0.2)	(0.222)	(0.225)	(0.267)	(0.248)	(0.223)
σ_η	0.448	0.393	0.4	0.373	0.413	0.48
	(0.061)	(0.121)	(0.086)	(0.094)	(0.082)	(0.034)
σ_ϵ	0.45	0.25	0.212	0.237	0.297	0.172
	(0.059)	(0.15)	(0.139)	(0.127)	(0.106)	(0.132)
σ_ξ	0.212	0.383	0.368	0.237	0.205	0.375
	(0.127)	(0.137)	(0.123)	(0.148)	(0.119)	(0.106)
Kalman gain(K)	0.54	0.808	0.813	0.678	0.657	0.717

This table shows the mean value and standard deviation of estimated coefficients obtained by SMM with time-varying θ model. Standard deviation is inside the brackets.